Teaching Mathematics *and* English to English Language Learners Simultaneously

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Oksana, born in the Ukraine, has been in the United States for more than a year. Her family lives in a new development in town, and her parents speak very little English. Like her three younger siblings, she has made friends, but tends to interact only when approached. While her English skills are improving and her hard work is evident, she experiences a great deal of frustration in the mathematics classroom. In the past, Oksana was asked to solve problems and give the answer, often told exactly how to arrive at the solution. Now, to succeed, Oksana not only has to provide the right answer, but she has to explain her reasoning and the process by which she arrived at her solution. Oksana is sometimes confused, sometimes frustrated, but always challenged by this new classroom norm that requires sophistication on both mathematical and linguistic levels.

Oksana's situation is not unique. While English language learners (ELLs) have varying levels of mathematical proficiency, those in reform-oriented classrooms rich in problem solving, reasoning, and communication (National Council of Teachers of Mathematics, 2000) are faced with added challenges. Here, students are required to "talk math" (paraphrasing Lemke, 1990); that is, to use the specific academic language needed to learn and express mathematical knowledge. Talking math not only makes use of specialized vocabulary (e.g., "logarithmic," "asymptotic," and "collinear"), but it also uses a variety of words and phrases that mean one thing in mathematics and another in everyday contexts (e.g., "rational" and "circular"). Just as the use of formal algebraic symbols can be a barrier to some students who are first learning about algebraic concepts, conversation in mathematics classrooms can be a barrier to understanding for ELLs.

This article discusses ways in which middle school mathematics teachers can assist their students, and particularly ELLs, in a two-for-one learning experience—learning both mathematics and the discourse skills needed to successfully participate in reform-oriented mathematics classrooms. Much of the discussion can easily generalize to other content areas. We begin with a brief overview of the changing U.S. demographic picture and its impact on schools. We then focus on the unique characteristics of common mathematical discourse, the challenges it can pose to ELLs, and strategies that can help students learn to "talk math."

This article reflects the following *This We Believe* characteristics: An inviting, supportive, and safe environment — Curriculum that is relevant, challenging, integrative, and exploratory —Multiple learning and teaching approaches that respond to student diversity

Who are our English language learners?

Current census data indicate that school districts throughout the United States are increasingly serving a student population whose home languages and cultures are diverse. For the 1993–1994 school year, the National Clearinghouse for English Language Acquisition (NCELA) (2004) reported an ELL student enrollment of 3 million in the U.S. This increased to 5 million just 10 years later, representing approximately 10% of the student body. States with historically large percentages of ELLs (e.g., Arizona, California, Florida, Illinois, New York, and Texas) continue to show increases in this student population. However, current data also show large and unexpected growth of school-aged ELLs in states that have historically reported low numbers (e.g., Tennessee, Indiana, Georgia, Nebraska, North Carolina, & South Carolina).

English language learners are very different from each other as well as from native speakers; the above description of Oksana is but one example. Some ELLs come to the United States having attended school regularly and bring with them literacy skills and content knowledge, although in another language. For these students, the transition into an academic setting in a second language will likely be far easier than for students who may come with a history of survival within a war-torn country or from an area where schooling was not always available or accessible. Many will belong to very low-income families, although their parents may be highly educated and once held professional positions. Some will speak a degree of English, and some will speak no English at all. For all learners, it is imperative that we find out "who our students are" so that we can appreciate and build on the resources they bring and better understand their needs.

Fortunately, in spite of these differences, there are needs all ELLs share. In addition to having to build their oral English skills, they also need to acquire reading and writing skills in English, while at the same time keeping up their learning in all content areas, including mathematics. Some ELLs will have other compounding needs. For example, many students (e.g., Chinese and Arabic speakers) might not be familiar with the Latin alphabet, and some students will have no familiarity with using letters of any kind. Difficulty in changing to the Latin alphabet can be compounded by the common use of these same symbols in algebraic and other mathematical representations. Regardless of these differences, and as stated by the National Council of Teachers of Mathematics (1994) position on language minority students:

> Cultural background and language must not be a barrier to full participation in mathematics programs preparing students for a full range of careers. All students, regardless of their language or cultural background, must study a core curriculum in mathematics based on the NCTM standards. (p. 20)

Thus, the NCTM standards have special implications for mathematics teachers working with ELLs. As the number of ELLs continues to increase, mathematics teachers increasingly need to consider two-for-one teaching strategies that address both the language and mathematical learning needs of these students. The recently published *TESOL PreK–12 English Language Proficiency Standards* (Teachers of English to Speakers of Other Languages, 2006) also provide specific recommendations for developing English language learners' oral language and literacy through academic content. Below we discuss some of the unique characteristics of the specialized language needed to talk math and then provide an assortment of teaching strategies for learning and talking mathematics.

The specialized language of mathematics

The development of the *math register,* that is, the language used to talk about mathematics, is a critical component of developing mathematical understanding in all students, including ELLs. But research on language acquisition suggests that mathematical discourse and representations have features that make it difficult for ELLs to draw meaning (Moschkowitz, 2000; Secada, Fennema & Adajian,

1995). The use of specialized symbols, technical language, and the various ways mathematical operations can be represented are a few of these features. A discussion of language, vocabulary types, and the unique grammar used in the teaching and learning of mathematics is presented in this section.

Differences between social and academic language

Social language is used in everyday, face-to-face interactions. While this is the speech most used during recess, in the hallway, and outside the school, it is also much needed in the classroom. ELLs generally acquire social language in two years (Cummins, 2005). Academic language, on the other hand, is used to acquire new knowledge or skills, develop deeper understanding of a topic, and communicate that understanding to others; it is the language students must use to effectively participate in content-rich discourse. There is evidence (Cummins, 2005; Scarcella, 2003) that the acquisition of academic language and literacy skills needed to fully participate in the mathematics classroom can take five to seven years. Obviously, distinctions between social and academic language are not precise, as classroom discourse (and even mathematical discourse) often makes use of both. Because of this, Moschkowitz (2000) points out that ELLs not only translate between English and their home language, but between both sets of social and academic languages. Thus, mathematics teachers who value communication in the classroom must consider an ELL's ability to participate in both "everyday" and "mathematical" kinds of interactions.

Mathematical vocabulary

The math register includes a variety of words, phrases, and expressions, which can be placed into four categories (Figure 1). *High-frequency vocabulary,* learned in almost any setting, overlaps social and academic language and consists of words and phrases most commonly heard and used. *General vocabulary,* primarily learned in classrooms and more formal settings, allows students to communicate about specific topics.

In addition to these more general kinds of words and phrases, mathematics teachers must also develop *specialized* and *technical vocabulary,* which are words and phrases specific to the mathematical content under discussion. Wong-Fillmore & Snow (2000) have listed a series of words that pose many challenges for ELLs, such as terms that express various kinds of quantitative relationships as well as everyday words that provide logical links in sentences typical to mathematical word problems (Figure 2).

Representing information in non-linguistic ways is also an important consideration when "talking math." For example, the idea of slope can be expressed using graphs of lines, algebraic symbols and formulas, tables of values, or with contextual information (e.g., the fixed cost of an item is the slope of a cost function for that specific item). Further, there are a variety of linguistic expressions commonly used to refer to the general concept of slope, including "rate of increase/decrease/change," "linear change," "degree of inclination," and "rise over run." Students must draw from all four of the vocabulary types when participating in mathematical conversations of this kind. As all teachers of mathematics

Figure 1 Vocabulary types common in mathematics classrooms *(adapted from Ernst-Slavit and Slavit, 2007).*

Type	Description	Examples
High-frequency vocabulary	Mostly social language; Terms used regularly in everyday situations	small, orange, clock
General vocabulary	Mostly academic language; Terms used in school but not directly associated with mathematics	combine, describe, consequently
Specialized vocabulary	Academic language; Terms broadly associated with mathematics	number, angle, equation, average
Technical vocabulary	Academic language; Terms associated with a specific mathematical topic	perfect numbers, supplementary angles, quadratic equations, cosine, mode

Figure 2 Problematic words for ELLs commonly used in mathematics textbooks and classrooms

Words that express quantitative relationships	Words that link phrases and sentences and express a logical relationship
hardly, scarcely, rarely, next, last, most, many, less, longer, older, younger, least, higher	if, because, unless, alike, same, different from, opposite of, whether, since, unless, almost, probably, exactly, not quite, always, never

know, specific language considerations are also needed due to the precise meaning of mathematical terms; for example, slope is a "rate of change," but not all rates of change have a slope. Hence, ELL students need to be made especially aware when their language can be "loose," and when it must be precise.

Mathematical grammar

At the sentence level, there are language patterns and grammatical structures specific to mathematics. These include the use of logical connectors (e.g., "consequently," "however") that in regular usage signal a logical relationship between parts of a text, but in mathematics signal similarity or contradiction. Likewise, the use of comparative structures (e.g., "greater than" and "less than," "n times as much as") and prepositions (e.g., "divided *by*," "divided *into*") pose serious difficulties for students who are trying to learn the content while, at the same time, trying to learn the language used to access that content. Semantic aspects of language can also pose difficulties, as in the following example (Dale & Cuevas, 1992): *Three times a number is 2 more than 2 times the number. Find the numbers.*

Solving this problem requires a recognition of how many numbers are involved, the relationships between them, and which ones need to be identified. In addition, ELLs (Dale and Cuevas, 1992) and other students (Clement, 1982) often encounter difficulties when they attempt to read and write mathematical sentences in the same way they read and write narrative text. That is, students may try to literally translate a mathematical concept expressed in words into a concept expressed in symbols. For example, the algebraic phrase "the number a is five less than the number b" is often translated into a=5-b, when it should be a=b-5.

Teaching strategies for learning and talking mathematics

The above discussion was intended to provide information to assist teachers to better understand the language abilities and needs of their learners. But we still must ask, "How do mathematics teachers teach their students mathematical thinking if their students speak very little English?" Or, as we are often asked, "How do I reach my ELLs?" Although there is no simple answer for this question, the truth is that in many mathematics classrooms, teachers are using a variety of instructional strategies that have proven useful for reaching all students, but, in particular, those who are learning English as a second language (see Figure 3 for some general approaches currently in use). It is important to remember the demands being placed on ELLs in these learning situations:

> ELLs are doing two jobs at the same time: learning a new language while learning new academic content. ELLs are moving between the two worlds of their ESL classroom and their content classrooms, and they have to work harder and need more support than the average native English-speaking student who has an age-appropriate command of the English language. (Carrier, 2005, p. 6)

Lee and Fradd (Lee & Fradd, 1998, 2001; Lee, 2004) have articulated a specific framework for assisting students in the context of science. This "instructional congruence framework" emphasizes the integration of students' language and cultural experiences with content and literacy development. Such an approach should emphasize the many cultural and linguistic strengths that ELLs bring to a learning situation (Ernst-Slavit & Slavit, 2007). Johnson (2005) showed how this could be

used in a multicultural seventh-grade classroom studying bioterrorism. Johnson's students drew from their political experiences, knowledge of air-borne diseases from their native countries, and stories of family members to eventually author a student handbook for responding to a bioterror event. Such a student experience is directly in line with the instructional congruence framework and employs a two-for-one teaching strategy aimed at both linguistic and content development.

Thompson and Rubenstein (2000) provided a general list of strategies for teaching mathematical vocabulary. We build on and expand that list to provide selected strategies that support ELLs as they learn mathematics, the math register, and how to "talk math."

Introduce new vocabulary in a thoughtful and integrated manner

Vocabulary is best taught not as a separate activity, but as part of the lesson. For example, students who memorize the definition of "square" without solving a problem or having discussion involving squares often make superficial meaning of this term. Manipulatives and visual aides, such as pictures, graphic organizers, charts, and bulletin boards, are good support for these conversations. It has been recommended that the introduction of new vocabulary be limited to fewer than 12 words per lesson (Fathman, Quinn, & Kessler, 1992). In addition, teachers can better communicate with their ELLs if they limit the use of idioms, speak slow, and use visuals and gestures. Breaking the lesson into smaller units and pausing and stressing key terms is also helpful.

English language learners benefit from teaching strategies that help them learn the specialized language of mathematics.
photo by Alan Geho

Identify and highlight key words with multiple meanings

In addition to the problematic words and phrases discussed above, ELLs can have difficulty with words that have multiple meanings in social and academic language, or in other content areas. For example, the word "table" can refer to a "times table" for multiplication facts or a "table of values" for graphing functions. "Table" may also have very different meanings and usages in non-mathematical contexts such as "timetable" in social studies, "table of contents" in language arts, "water table" in physical science, and "periodic table" in chemistry. Identifying and carefully planning the use of any such words in a lesson can support students' efforts to follow the subsequent line of discourse.

Preview and review

This technique provides a lesson introduction (which can be given to all students or only to ELLs) via a handout, an outline of the entire lesson on the board or overhead, and a list of key words. This preview provides context for the lesson, and small-group discussion can support any of these steps. After the lesson, a review of its main aspects, including both key content and language features, can be provided to further clarify or reinforce learning goals as well as key terms. Handouts or small-group discussion could be used for this step as well.

Kristie, a middle school science teacher, makes use of this technique in all of her classes, including those with ELLs. Her use of preview is extended through the use of a "hula skirt," a piece of paper folded down the middle and cut horizontally into four or five strips on each side. The students are asked to write key terms and definitions on the left and provide a visual on the right. These terms are then used during the lesson, and the students make regular use of the hula skirt throughout. Kristie states:

> For my ELLs, I always try to use different modalities to get them to understand the vocabulary. The hula skirt is kind of fun, and it gets them to write a definition and connect it to a visual. I tell them I am bad at Pictionary, you know, like stick figures and stuff, so the drawing doesn't have to be perfect. But it really connects them to the meaning of the word.

Kristie also uses the hula skirt for a Jeopardy-like game, by having one of a pair of students fold and cover the strip with the illustration, and asking the other to provide either the word or definition.

Brainstorming the meaning and origin of technical terms

Helping students brainstorm the meaning of technical words and expressions might unveil potential connections between the meaning of the word, the student's language background, and the math register. For example, discussing "degrees" as the amount it "grades out" the circular distance between the angle's rays can connect this term and idea to the Spanish word "grados." Word origins and relationships can also be helpful, such as discussing how the term hypotenuse is derived from the Greek word for "stretching under," or connecting the word "rational" to "ratio" to help make clear that all rational numbers can be expressed as a ratio of integers.

Figure 3 Program models and approaches for teaching mathematics (and other content) to English language learners

Title	Approach	Description
Content-based language instruction (TESOL, 2006)	Teachers use and adapt materials from the mathematics curricula as a vehicle for developing language and content.	Also called *integrated language and content instruction,* it is usually taught by a specialist in both mathematics and ESL.
Sheltered instruction (TESOL, 2006)	The mathematics curricula is adapted to accommodate students' level of English language proficiency.	Teachers use mathematical materials that are challenging, but may be at a lower reading level. Abstract concepts are broken down to concrete attributes, and vocabulary skills are enhanced. It serves to transition students from the ESL class to the academic mainstream.
Specially-Designed Academic Instruction in English (SDAIE) (Diaz-Rico & Weed, 2006)	Teaching of grade-level subject matter in English while also promoting English language development.	Teachers are encouraged to focus on (a) their own speech, by limiting the use of idioms, speaking slowly, and using everyday meaningful vocabulary; (b) the use of visuals and contextual clues, including gestures to convey meaning; and, (c) lesson planning that uses and builds on students' background knowledge.
Guided Language Acquisition Design (GLAD) www.projectglad.com/	Actively using language across content areas.	Lessons are planned around (a) an engaging topic; (b) motivation; (c) multiple forms of review and evaluation; and (d) specific vocabulary, concepts, skills, and higher order thinking skills.
Optimal Learning Environment (OLE) (Ruiz & Figueroa, 1995)	Outlines specific conditions that promote student learning of content.	These include high expectations, immediate feedback, building community, placing meaning before form, and immersing students in print.
The Sheltered Instruction Observational Protocol (SIOP) (Echevarria, Vogt, & Short, 1999)	Promotes self-evaluation and reflection.	The protocol provides extensive criteria for effective planning and instruction. Emphasizes clear content and language objectives, building background knowledge, promoting interaction, practice, application, and assessment.
Cognitive Academic Language Learning Approach (CALLA) (Chamot & O'Malley, 1994)	Integrates content-area and language instruction with explicit attention to learning strategies.	Based on cognitive learning theory, CALLA emphasizes skills that promote active learning.

Validating students' languages and cultures

Research indicates that students' home languages can play a significant role in learning complex material, including content encountered in mathematics classrooms. This is especially true when students are afforded opportunities to incorporate their home languages into classroom discourse (Thomas & Collier, 2002). Even teachers who do not speak an ELL's home language can still make use of this strategy by affording opportunities for students to access books, handouts, or Web sites in their native language, or working with a peer or teaching assistant versed in the native language.

Arthur, a middle school teacher in a building with a large number of Mexican and Central American students, builds on students' knowledge of Spanish by using cognates—a word in one language that is similar in meaning and form to a word in another language. Arthur states, "My Spanish-speaking students understand more English than they realize. For example, they know círculo (circle), lateral (lateral; related to the side), cuadrado (a square or special quadrilateral), and even words like edificio (edifice), casi (quasi; resembling something), and creciendo (crescendo)." The use of cognates helps Arthur validate the students' first language while enabling students to learn language and content through vocabulary that can be easily identifiable in its written form.

All students come with varied lived experiences and knowledge that often leads to creative ways of solving mathematical problems. Sharing such samples of student thinking and problem solving is currently at the heart of mathematics education reform. But classrooms with ELLs can be endowed with unique perspectives on concepts or algorithms learned in another school culture or perhaps through a novel context for application of a specific mathematical topic. Such contributions could be organized through the use of writing activities or in small groups, as discussed next.

Cooperative learning and other opportunities for interaction

It is possible for students of diverse linguistic and educational backgrounds to work together on a common task in pursuit of a common goal. Collaborative groups provide opportunities for students to hear and use the math register while also developing mathematical understanding. Depending on the students' language proficiency, this works very well in groups with diverse language backgrounds, since students must use English to communicate with all the members of the group. Teachers can provide visuals with key words to support students with emerging language proficiency, even in groups with a variety of home languages.

Maddie, a seventh grade mathematics teacher with students from Mexico, Eastern Europe, and Africa, made simultaneous use of several two-for-one strategies when she asked her class to count on their fingers. Maddie noticed that Chimwala began counting with her thumb, others began with their pinky, and most with their index fingers. After this realization, Maddie asked all her students to share in groups how they use their fingers, or any other body parts, in the counting process. Though she did not choose to do so, Maddie could have extended this discussion into exploring the various algorithms for performing arithmetic on whole numbers that students bring from their various home and school cultures.

> **"Collaborative groups provide opportunities for students to hear and use the math register while developing mathematical understanding."**

Taking risks and making mistakes

Learning a second language, including the math register, has an affective base. Students need to be encouraged to ask questions and take risks; making mistakes is a part of learning. If students' answers are not correct, or if students are not able to follow the emerging lines of discourse, patience may be needed to ensure that student risk-taking and participation will continue.

Conclusions

All students need support to participate in mathematical conversations, but attention to equity in a mathematics classroom must address the linguistic demands placed on ELLs; mathematical discourse is not easily accessible when presented in a second language. Learning the math register can become a complex endeavor for ELLs, because many words cannot be translated from English to their native languages, and comparable terms and parallel ways of considering ideas may not exist across languages (Lee & Fradd, 1998). This article has offered a perspective for thinking about the role of language in mathematical development, and ways in which teachers of mathematics can facilitate the two-for-one learning goal of content and linguistic development. Though not always easy to implement, the above strategies can enrich the mathematical learning experience for all students, including English language learners.

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